Memristor – The Missing Circuit Element

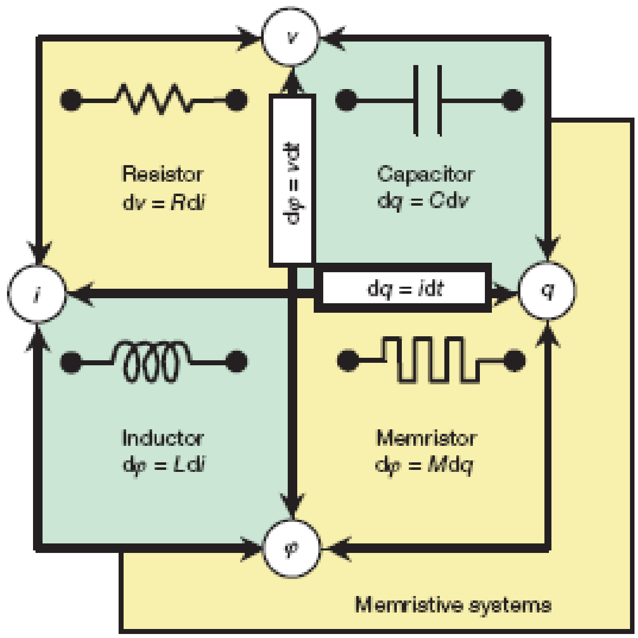


Figure 1 | The four fundamental two-terminal circuit elements: resistor, capacitor, inductor and memristor. Resistors and memristors are subsets of a more general class of dynamical devices, memristive systems. Note that R, C, L and M can be functions of the independent variable in their defining equations, yielding nonlinear elements. For example, a charge-controlled memristor is defined by a single-valued function $M(q)$.

Basic definition of a memristor (Chua, 1971)

$$g(\varphi, q) = 0$$

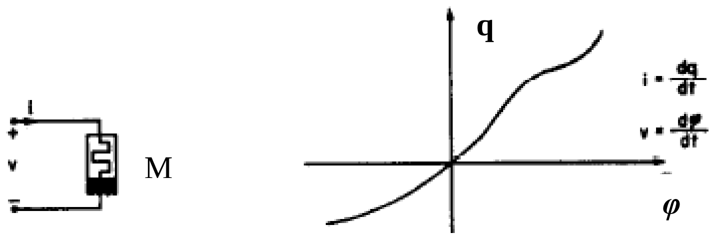


Fig. 2. The symbol used for a memristor and a generic (flux-controlled) characteristic

Note that the flux $\varphi(t)$ associated with a two-terminal element at any time *t* is defined by

$$\varphi(t) = \int_{-\infty}^{t} v(\tau)d\tau$$

without any implication with magnetic field.

If the implicit equation above can be put in the explicit form

$$\varphi = f(q)$$

then we get the constitutive equation of a charge-controlled memristor. In this case we have

$$d\varphi = M(q)dq, \text{ where } \quad M(q) = \frac{df}{dq}$$

The voltage across a charge-controlled memristor is given by

$$\begin{cases} v(t) = \dfrac{d\varphi}{dt} = \dfrac{df}{dq}\dfrac{dq}{dt} = M(q(t))i(t) \\ \dfrac{dq}{dt} = i(t) \end{cases}$$

At any time t_0, M depends on $q(t_0)$, i.e. on the time integral of the memristor current

$$q(t_0) = \int_{-\infty}^{t_0} i(\tau)\, d\tau$$

and, hence, it depends on the complete past history of the memristor current. This observation justifies the choice of the name *memory resistor*, or *memristor*.

Finally, note that there is no point introducing a linear memristor in linear network theory. In fact, in this case we would have

$$\frac{df}{dq} = M = \mathrm{const}$$

and, hence, the memristor would reduce to a linear resistor

$$v(t) = M\, i(t)$$

General memristive one-port (Chua, Kang, 1976)

General expression for a current-controlled memristive one-port (Chua, Kang, 1976):

$$\begin{cases} v = R(w)i \\ \dfrac{dw}{dt} = f(w,t) \end{cases}$$

where w denotes the state of the system.

Note that the output v is zero whenever the input i is zero,

regardless of the state w, which incorporates the memory effect.

Main properties:

1) Passivity criterion
2) No energy discharge property
3) DC characteristics
4) Double-valued Lissajous figure property:

A current-controlled memristive one-port under periodic operation with $i(t) = I\cos(\omega t)$ always gives rise to a v-i Lissajous figure whose voltage v is at most a double-valued function of i.

Proof: In the representation above, the state equation has a unique periodic solution $x(t)$ for all $t \geq t_0$ for some initial state x_0, by assumption. Hence, for any value of the current $i \in [-I, I]$, there correspond at most two distinct values of v.

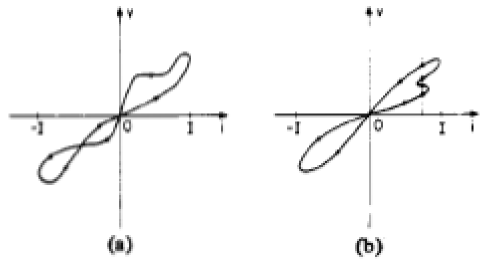
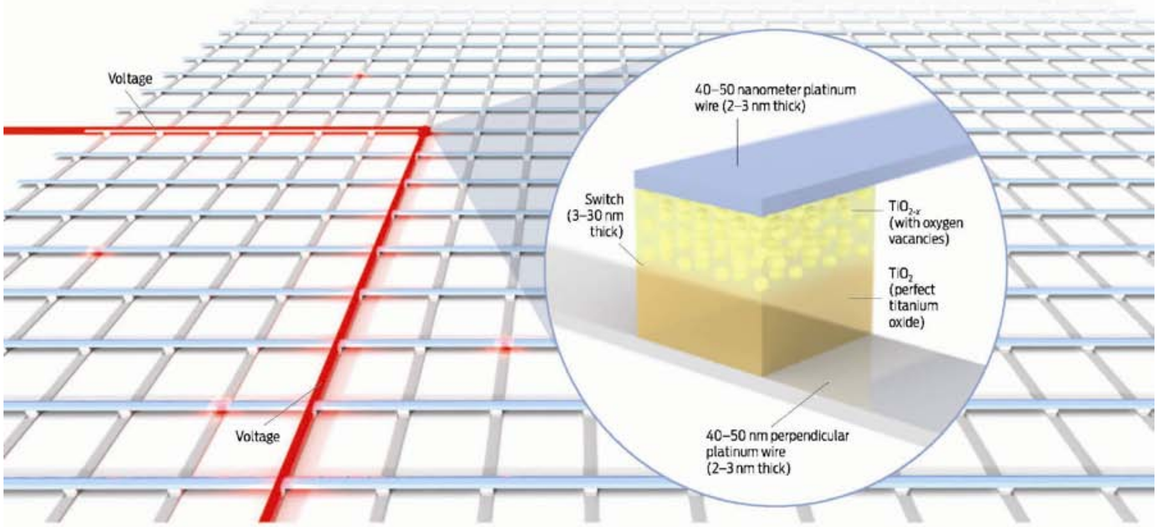


Fig. 5. Illustration of property 4. (a) Possible Lissajous figure. (b) Impossible Lissajous figure.

Remark: This property is illustrated in Fig. 5. Observe that the Lissajous figure in Fig. 5(b) cannot correspond to that of a current-controlled memristive one-port, because at $i = i_P$, there correspond more than two distinct values of v.

► The physical circuit

How Memristance Works



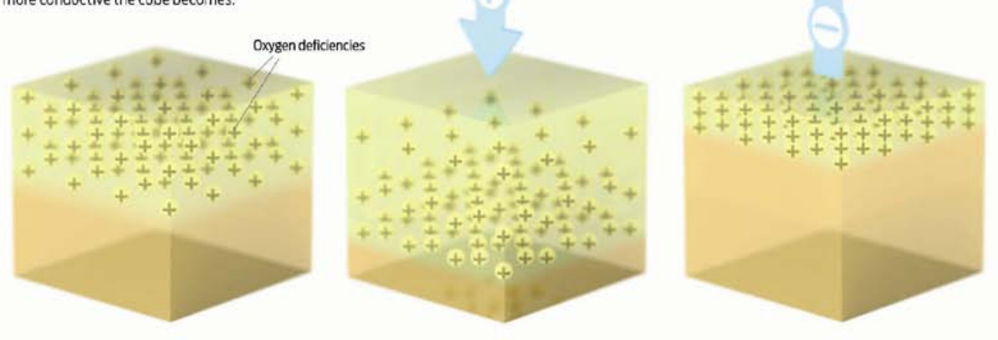
THE CROSSBAR ARCHITECTURE: The crossbar architecture is a fully connected mesh of perpendicular wires. Any two crossing wires are connected by a switch. To close the switch, a positive voltage is applied across the two wires to be connected. To open the switch, the voltage is reversed.

THE SWITCH: A switch is a 40-nanometer cube of titanium dioxide (TiO_2) in two layers: The lower TiO_2 layer has a perfect 2:1 oxygen-to-titanium ratio, making it an insulator. By contrast, the upper TiO_2 layer is missing 0.5 percent of its oxygen (TiO_{2-x}), so *x* is about 0.05. The vacancies make the TiO_{2-x} material metallic and conductive.

APPLIED MEMRISTANCE: The oxygen deficiencies in the TiO_{2-x} manifest as "bubbles" of oxygen vacancies scattered throughout the upper layer. A positive voltage on the switch repels the (positive) oxygen deficiencies in the metallic upper TiO_{2-x} layer, sending them into the insulating TiO_2 layer below. That causes the boundary between the two materials to move down, increasing the percentage of conducting TiO_{2-x} and thus the conductivity of the entire switch. The more positive voltage is applied, the more conductive the cube becomes.

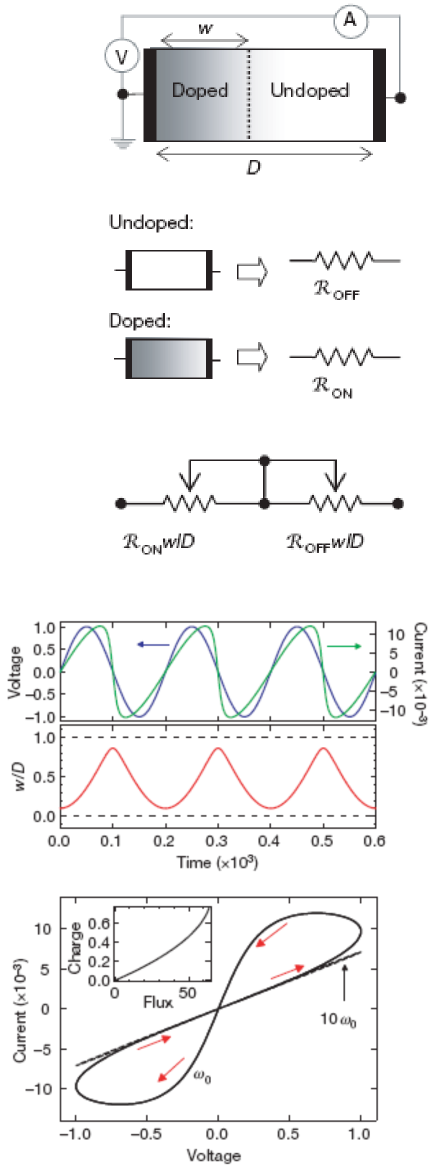
A negative voltage on the switch attracts the positively charged oxygen bubbles, pulling them out of the TiO_2. The amount of insulating, resistive TiO_2 increases, thereby making the switch as a whole resistive. The more negative voltage is applied, the less conductive the cube becomes.

What makes this switch special—memristive—is that when the voltage is turned off, positive or negative, the oxygen bubbles do not migrate. They stay where they are, which means that the boundary between the two titanium dioxide layers is frozen. That is how the memristor "remembers" how much voltage was last applied.



BRYAN CHRISTIE DESIGN

The model



The application of an external bias $v(t)$ across the device will move the boundary between the two regions by causing the charged dopants to drift[26]. For the simplest case of ohmic electronic conduction and linear ionic drift in a uniform field with average ion mobility μ_V, we obtain

$$v(t) = \left(\mathcal{R}_{ON}\frac{w(t)}{D} + \mathcal{R}_{OFF}\left(1-\frac{w(t)}{D}\right)\right)i(t) \qquad (5)$$

$$\frac{dw(t)}{dt} = \mu_V \frac{\mathcal{R}_{ON}}{D} i(t) \qquad (6)$$

which yields the following formula for $w(t)$:

$$w(t) = \mu_V \frac{\mathcal{R}_{ON}}{D} q(t) \qquad (7)$$

By inserting equation (7) into equation (5) we obtain the memristance of this system, which for $\mathcal{R}_{ON} \ll \mathcal{R}_{OFF}$ simplifies to:

$$\boldsymbol{R(q)} \equiv \quad M(q) = \mathcal{R}_{OFF}\left(1-\frac{\mu_V \mathcal{R}_{ON}}{D^2}q(t)\right) \qquad (*)$$

The q-dependent term in parentheses on the right-hand side of this equation is the crucial contribution to the memristance, and it becomes larger in absolute value for higher dopant mobilities μ_V and smaller semiconductor film thicknesses D. For any material, this term is 1,000,000 times larger in absolute value at the nanometre scale than it is at the micrometre scale, because of the factor of $1/D^2$, and the memristance is correspondingly more significant. Thus, memristance becomes more important for understanding the electronic characteristics of any device as the critical dimensions shrink to the nanometre scale.

(*) $$v(t) = \left[R_{ON}\mu_v \frac{R_{ON}}{D^2}q + R_{OFF}\left(1-\mu_v\frac{R_{ON}}{D^2}q\right)\right]i(t) = R_{OFF}\left[\mu_v\frac{R_{ON}^2}{R_{OFF}}\frac{1}{D^2}q + \left(1-\mu_v\frac{R_{ON}}{D^2}q\right)\right]i(t)$$

If $R_{ON} \ll R_{OFF}$ the term R_{ON}^2 / R_{OFF} can be disregarded and hence:

$$\begin{cases} v(t) = R(q)i(t) \quad \text{with } R(q) \equiv M(q) = R_{OFF}\left[1-\mu_v\dfrac{R_{ON}}{D^2}q\right] \\ \dfrac{dq}{dt} = i \end{cases}$$