NONLINEAR DYNAMICS AND CHAOS

With Applications to Physics, Biology, Chemistry, and Engineering

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LINEAR AND NONLINEAR CIRCUITS

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Second, the dynamics of a nonlinear system are much richer than the dynamics of a linear system. There are "essentially nonlinear phenomena" that can take place only in the presence of nonlinearity; hence they cannot be described or predicted by linear models. Examples of essentially nonlinear phenomena are

- *Finite escape time*: The state of an unstable linear system goes to infinity as time approaches infinity; a nonlinear system's state, however, can go to infinity in finite time.

- *Multiple isolated equilibria*: a linear system can have only one isolated equilibrium point; hence it can have only one steady-state operating point which attracts the state of the system irrespective of the initial state. A nonlinear system can have more than one isolated equilibrium point. The state may converge to one of several steady-state operating points, depending on the initial state of the system.

- *Limit cycles*: For a linear time-invariant system to oscillate, it must have a pair of eigenvalues on the imaginary axis, which is a nonrobust condition that is almost impossible to maintain in the presence of perturbations. Even if we do, the amplitude of oscillation will be dependent on the initial state. In real life stable oscillation must be produced by nonlinear systems. There are nonlinear systems which can go into an oscillation of fixed amplitude and frequency, irrespective of the initial state. This type of oscillation is known as a limit cycle.

- *Subharmonic, harmonic, or almost-periodic oscillations*: A stable linear system under a periodic input produces an output of the same frequency. A nonlinear system under periodic excitation can oscillate with frequencies which are submultiples or multiples of the input frequency. It may even generate an almost-periodic oscillation, an example of which is the sum of periodic oscillations with frequencies which are not multiples of each other.

- *Chaos*: A nonlinear system can have a more complicated steady-state behavior that is not equilibrium, periodic oscillation, or almost-periodic oscillation. Such behavior is usually referred to as chaos. Some of these chaotic motions exhibit randomness, despite the deterministic nature of the system.

- *Multiple modes of behavior*: It is not unusual for two or more modes of behavior to be exhibited by the same nonlinear system. An unforced system may have more than one limit cycle. A forced system with periodic excitation may exhibit harmonic, subharmonic, or more complicated steady-state behavior, depending upon the amplitude and frequency of the input. It may even exhibit a discontinuous jump in the mode of behavior as the amplitude or frequency of the excitation is smoothly changed.

CDK, Linear and Nonlinear Circu

Simple circuits Circuits containing nonlinear resistors have properties totally different from those which have only linear resistors. The following examples illustrate some of the differences.

Example 1 (nonlinear resistors can produce harmonics) Consider a sinusoidal voltage waveform,

$$v(t) = 2 \sin \omega t \text{ (in volts)} \qquad t \geq 0$$

where the constant ω is the angular frequency in radians per second, i.e., $\omega = 2\pi f$ where f is frequency in hertz. If the waveform is applied to a linear resistor of $10\,\Omega$, the current is $i(t) = 0.2 \sin \omega t$ (in amperes), $t \geq 0$.

Let us apply the same voltage waveform to a nonlinear resistor which has the v-i characteristic shown in Fig. 1.15a, where

$$i = \hat{i}(v)$$

We wish to determine the current waveform $i(t)$ for $t \geq 0$. In simple

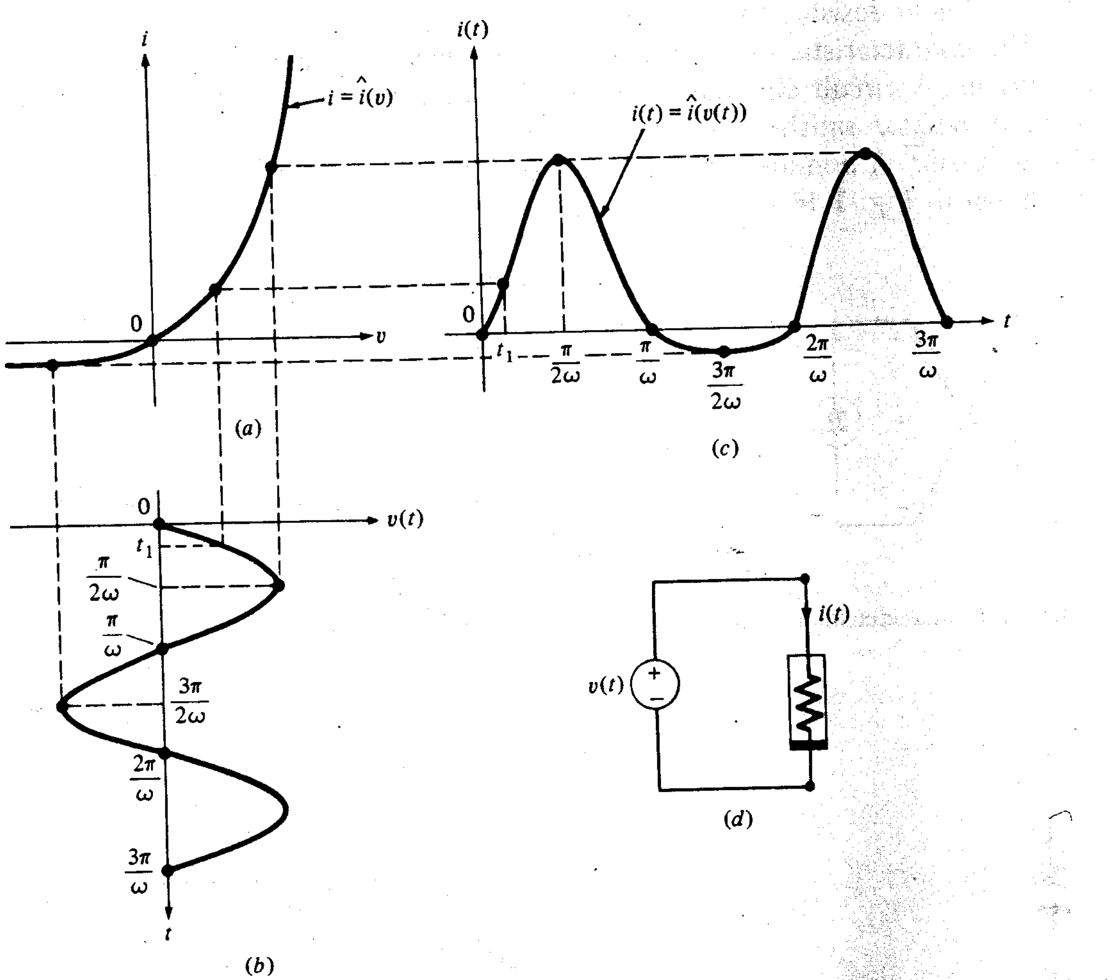

Figure 1.15 An example illustrating a special clipping property of nonlinear resistors; the negative half of the waveform has been clipped.