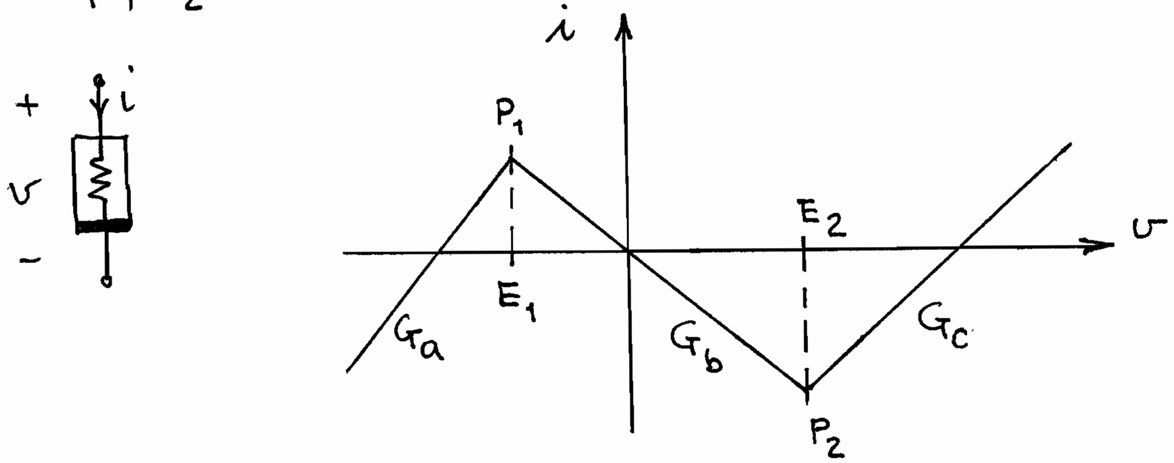
1) Si consideri il resistore non lineare indicato in figura.
Si scriva l'espressione analitica di tale caratteristica in funzione di v, G_a, G_b, G_c, E_1, E_2



Considero i come somma di tre contributi

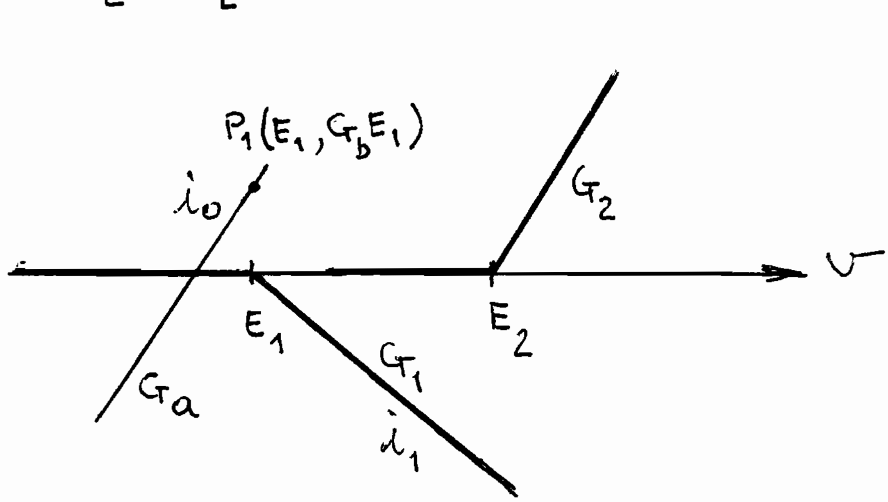
$$i = i_0 + i_1 + i_2$$

con:

$$i_0 = G_a(v - E_1) + G_b E_1 \quad (P_1 \text{ ha coordinate } E_1, G_b E_1)$$

$$i_1 = \frac{1}{2} G_1 \left[|v - E_1| + (v - E_1) \right] \quad (\text{concave resistor in } E_1)$$

$$i_2 = \frac{1}{2} G_2 \left[|v - E_2| + (v - E_2) \right] \quad (\text{concave resistor in } E_2)$$



deve essere:

$$G_a + G_1 = G_b$$

$$G_a + G_1 + G_2 = G_c$$

da cui si ottiene:

$$G_1 = G_b - G_a$$

$$G_2 = G_c - G_b$$

Determino l'espressione finale di i:

$$i = G_a(v - E_1) + G_b E_1 +$$
$$\frac{1}{2}(G_b - G_a)|v - E_1| + \frac{1}{2}(G_b - G_a)(v - E_1)$$
$$+\frac{1}{2}(G_c - G_b)|v - E_2| + \frac{1}{2}(G_c - G_b)(v - E_2)$$

$$i = E_1(G_b - G_a) + G_a v + \frac{1}{2}(G_b - G_a)|v - E_1| +$$
$$+\frac{1}{2}(G_b - G_a)v - \frac{1}{2}(G_b - G_a)E_1 + \frac{1}{2}(G_c - G_b)|v - E_2|$$
$$+\frac{1}{2}(G_c - G_b)v - \frac{1}{2}(G_c - G_b)E_2 ;$$

$$i = \frac{1}{2}(G_b - G_a)E_1 - \frac{1}{2}(G_c - G_b)E_2 +$$
$$+\left[G_a + \cancel{\frac{1}{2}G_b} - \frac{1}{2}G_a + \frac{1}{2}G_c - \cancel{\frac{1}{2}G_b}\right]v +$$
$$+\frac{1}{2}(G_b - G_a)|v - E_1| + \frac{1}{2}(G_c - G_b)|v - E_2|$$

$$i = \frac{1}{2}(G_b - G_a)E_1 - \frac{1}{2}(G_c - G_b)E_2 + \frac{1}{2}(G_a + G_c)v$$
$$+\frac{1}{2}\left[(G_b - G_a)|v - E_1| + (G_c - G_b)|v - E_2|\right]$$

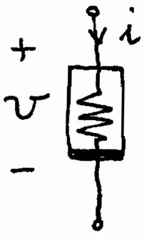
Nel caso particolare di caratteristica simmetrica rispetto all'origine:

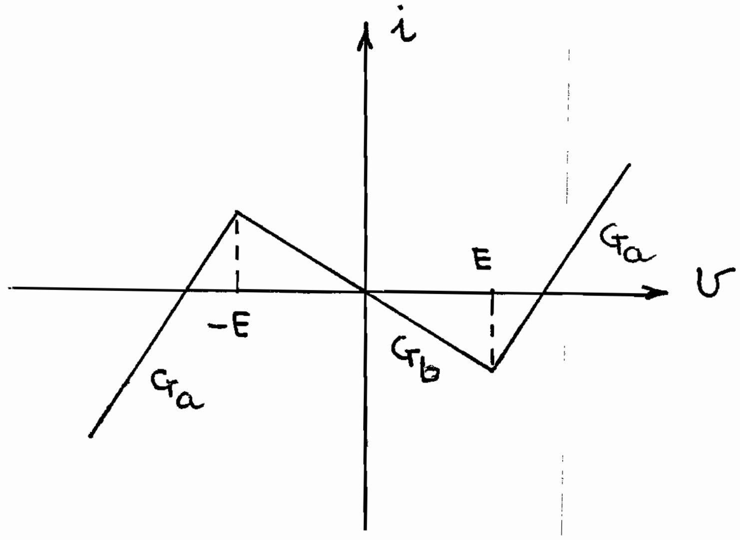
$$E_1 = -E \;,\; E_2 = E \;,\; G_a = G_c$$

si ottiene:

$$i = G_a v + \frac{1}{2}(G_b - G_a)\Big[|v+E| - |v-E|\Big]$$

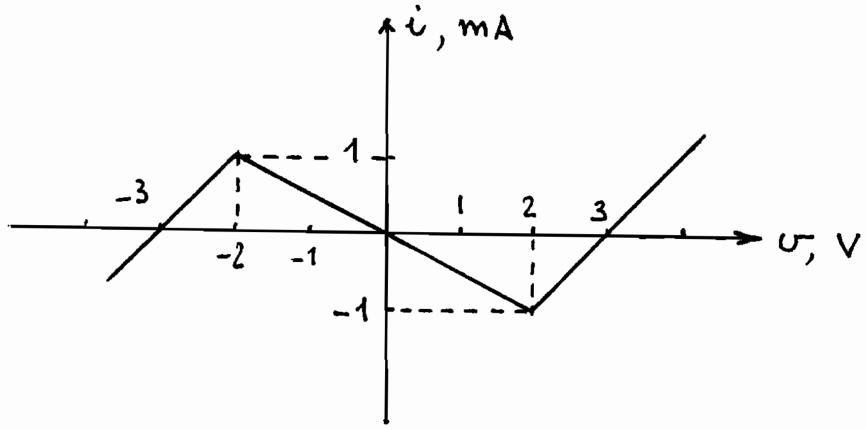
2) Si consideri il resistore non lineare la cui caratteristica è indicata in figura. Si scriva l'espressione analitica di tale caratteristica in funzione di v, G_a, G_b, E





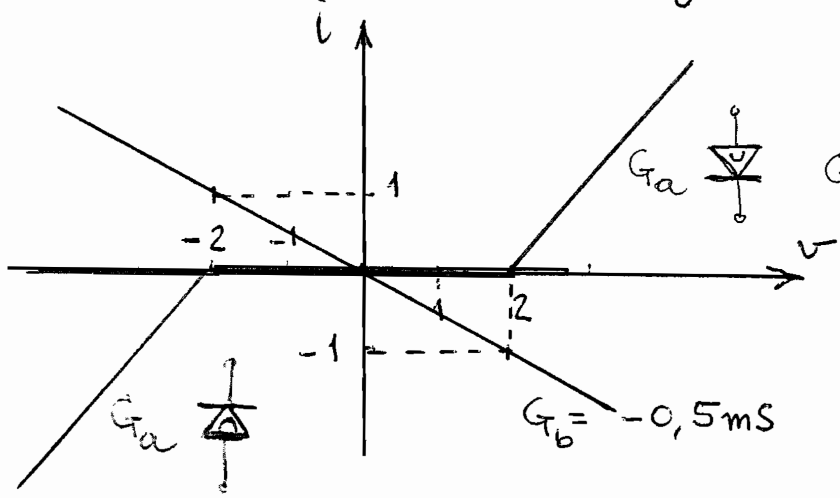
R.: $i = G_a v + \frac{1}{2}(G_b - G_a)\left(|v+E| - |v-E|\right)$

3) Usando solamente resistori concavi e un solo resistore lineare si sintetizzi un bipolo resistivo la cui caratteristica è data da:



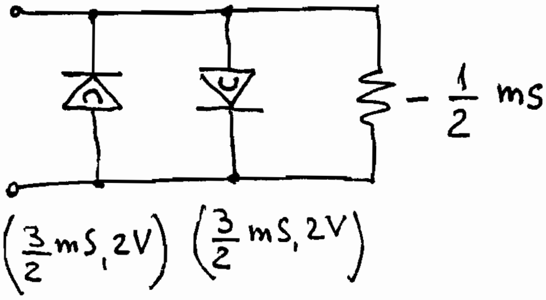
R.:

La caratteristica indicata si può considerare somma delle tre caratteristiche seguenti:

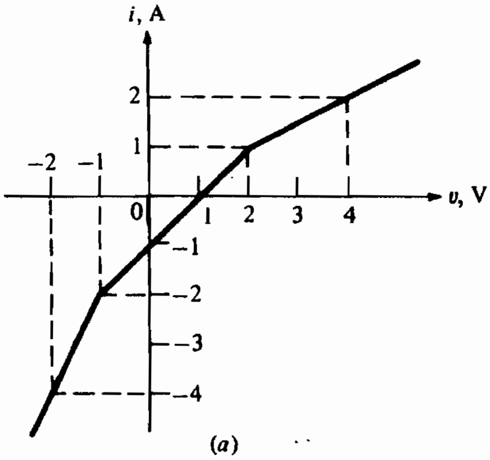


$G_a - 0,5 = 1 \Rightarrow G_a = 3/2\ mS$

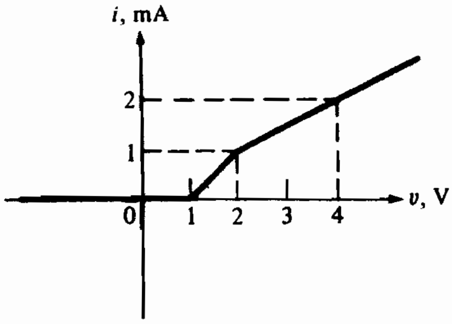
Il circuito finale è:



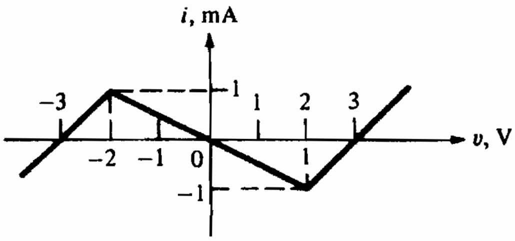
4) Si scrivano le espressioni analitiche delle caratteristiche lineari a tratti indicate in figura.



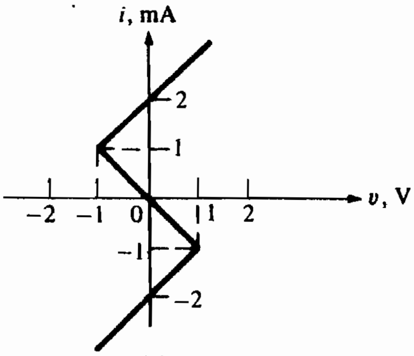
(a)



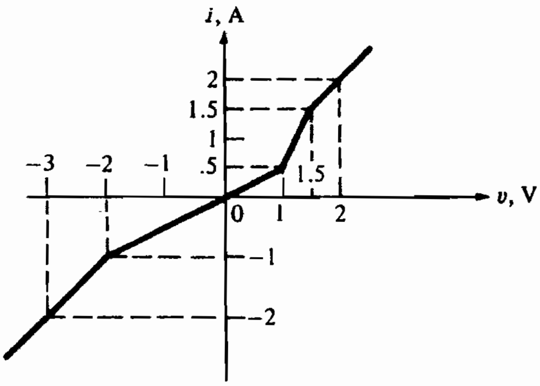
(b)



(d)



(c)



(e)

R:

a) $v = \frac{5}{4} i + \frac{1}{4} |i+2| + \frac{1}{2} |i-1|$

b) $i = \frac{1}{4} v + \frac{1}{2} |v-1| - \frac{1}{4} |v-2|$

c) $v = i - |i+1| + |i-1|$

d) $i = v - \frac{3}{4} |v+2| + \frac{3}{4} |v-2|$

e) $i = \frac{1}{2} + v - \frac{1}{4} |v+2| + \frac{3}{4} |v-1| - \frac{1}{2} |v-1.5|$

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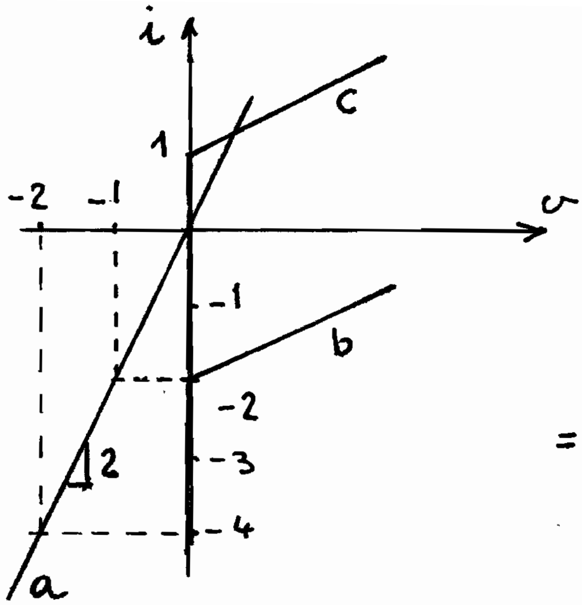
4 a)



$$\begin{cases} R_a = \frac{1}{2} \\ R_a + R_b = 1 \\ R_a + R_b + R_c = 2 \end{cases} \qquad \begin{cases} R_a = \frac{1}{2} \\ R_b = 1 - \frac{1}{2} = 1/2 \\ R_c = 2 - \frac{1}{2} - \frac{1}{2} = 1 \end{cases}$$

$$v = v_a + v_b + v_c =$$

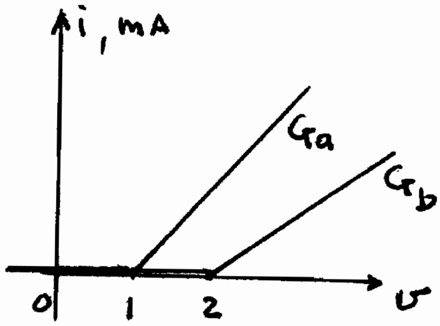
$$= \frac{1}{2} i + \frac{1}{4}\left[|i+2| + (i+2) \right] +$$

$$+ \frac{1}{2}\left[|i-1| + (i-1) \right] ;$$

$$v = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{2} \right) i \quad \cancel{+\frac{1}{2}} \cancel{-\frac{1}{2}} + \frac{1}{4}|i+2| + \frac{1}{2}|i-1| ;$$

$$v = \frac{5}{4} i + \frac{1}{4}|i+2| + \frac{1}{2}|i-1|$$

4 b)



$$\begin{cases} G_a = 1 \text{ mS} \\ G_a + G_b = \frac{1}{2} \text{ mS} \end{cases} \qquad G_b = -\frac{1}{2} \text{ mS}$$

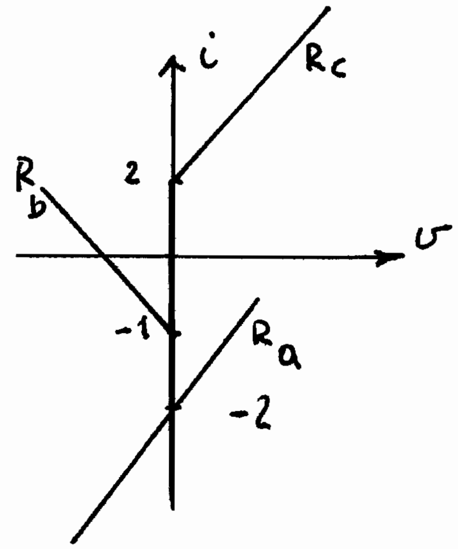
$$i = i_a + i_b = \frac{1}{2}\left[|v-1| + (v-1) \right] -$$

$$- \frac{1}{4}\left[|v-2| + (v-2) \right]$$

$$i = \left(\frac{1}{2} - \frac{1}{4} \right) v \quad \cancel{-\frac{1}{2}} \cancel{+\frac{1}{2}} + \frac{1}{2}|v-1| - \frac{1}{4}|v-2|$$

$$i = \frac{1}{4} v + \frac{1}{2}|v-1| - \frac{1}{4}|v-2|$$

4c)



$$v = v_a + v_b + v_c$$

$$v_a = R_a i - 2$$

$$\begin{cases} R_a = 1 \\ R_a + R_b = -1 \\ R_a + R_b + R_c = 1 \end{cases} \qquad \begin{cases} R_a = 1 \\ R_b = -2 \\ R_c = 2 \end{cases}$$

$$v = v_a + v_b + v_c = i - 2 + \left(-\frac{2}{2}\right)\Big[|i+1| + (i+1)\Big] + \Big[|i-1| + (i-1)\Big]$$

$$v = i - 2 - i - 1 + i - 1 - |i+1| + |i-1| = i + |i-1| - |i+1|$$

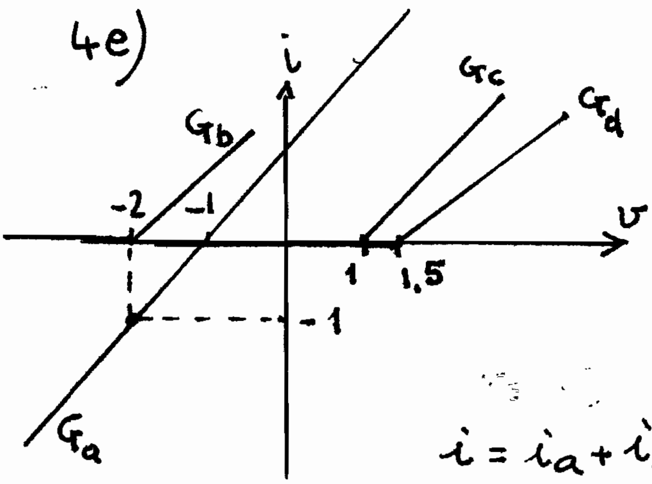
4d) si usino le formule dell'esercizio 2:

$$i = G_a v + \frac{1}{2}\left(G_b - G_a\right)\Big(|v+E| - |v-E|\Big)$$

$$G_a = 1 \quad G_b = -\frac{1}{2} \quad G_b - G_a = -\frac{3}{2} \quad E = 2$$

$$i = v - \frac{3}{4}|v+2| + \frac{3}{4}|v-2|$$

4e)



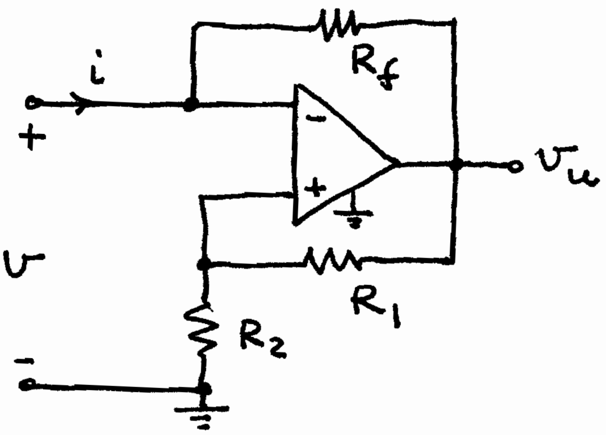
$$\begin{cases} G_a = 1 \\ G_a + G_b = 1/2 \\ G_a + G_b + G_c = 2 \\ G_a + G_b + G_c + G_d = 1 \end{cases} \qquad \begin{matrix} G_a = 1 \\ G_b = -\frac{1}{2} \\ G_c = 3/2 \\ G_d = -1 \end{matrix}$$

$$i = i_a + i_b + i_c + i_d = v + 1 - \frac{1}{4}\Big[|v+2| + (v+2)\Big] +$$
$$+ \frac{3}{4}\Big[|v-1| + (v-1)\Big] - \frac{1}{2}\Big[|v-1{,}5| + (v-1{,}5)\Big]$$

$$i = v\left(1 - \frac{1}{4} + \frac{3}{4} - \frac{1}{2}\right) + 1 - \frac{1}{2} + \frac{3}{4} - \frac{3}{4} - \frac{1}{4}|v+2| + \frac{3}{4}|v-1| - \frac{1}{2}|v-1{,}5|$$

$$i = \frac{1}{2} + v - \frac{1}{4}|v+2| + \frac{3}{4}|v-1| - 1/2\,|v-1{,}5|$$

5) Si ricavi la caratteristica $v \div i$ del bipolo indicato in figura, considerando ideale l'amplificatore operazionale.



In zona lineare il circuito presenta una resistenza negativa di valore

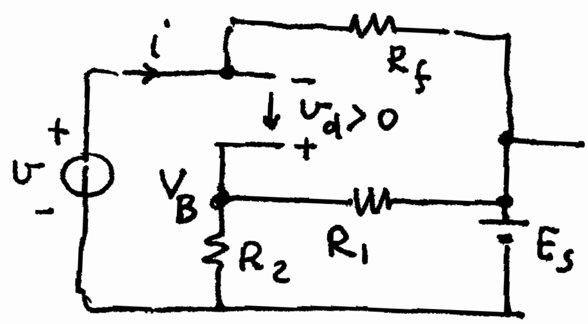
$$-\frac{R_2 R_f}{R_1}$$

Si rimane in zona lineare se $|v_u| \leq E_s$

Ora $v_u = (R_1+R_2)\dfrac{v}{R_2}$, quindi si è in zona lineare per:

$$|v| \leq \frac{R_2}{R_1+R_2} E_s = V_B$$

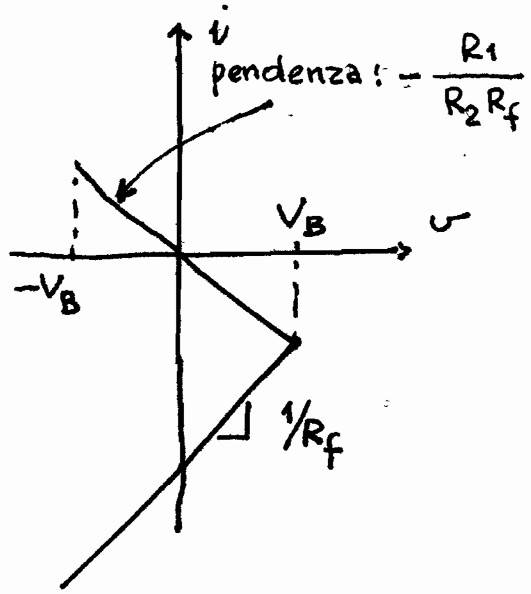
Quando vado in saturazione positiva, il circuito diventa



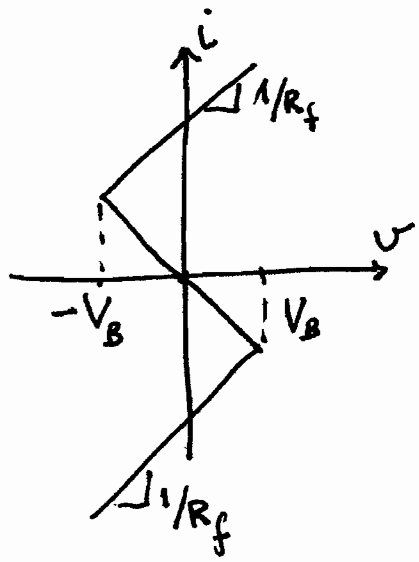
$i = \dfrac{v - E_s}{R_f}$, per $v_d > 0$

ovvero per $v < V_B$

Si ottiene:



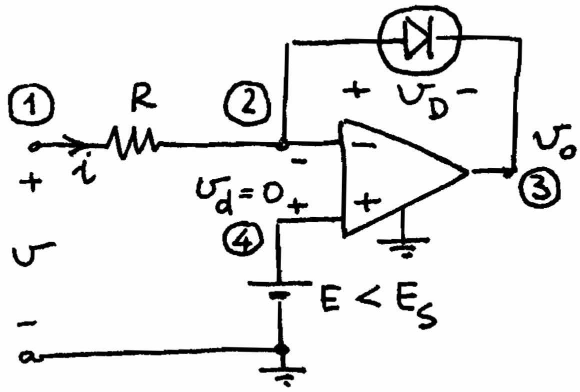
Cose analoghe per la zona di saturazione negativa. In conclusione la caratteristica è:



6) Regione lineare

Poichè l'amplificatore è ideale ($v_d = 0$), si ha

$$v = Ri + E \quad (1)$$



Si è in zona lineare se

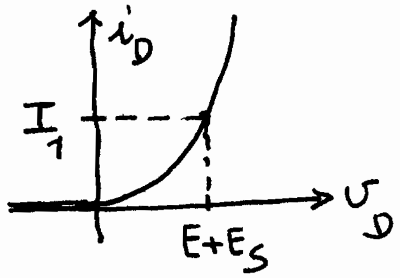
$$-E_S < v_o < E_S$$

Ma $v_o = E - v_D$

e quindi deve essere $-E_S < E - v_D < E_S$, ovvero

$$E - E_S < v_D < E + E_S$$

Di conseguenza, i_D deve essere inferiore al valore I_1 indicato in figura. Poichè



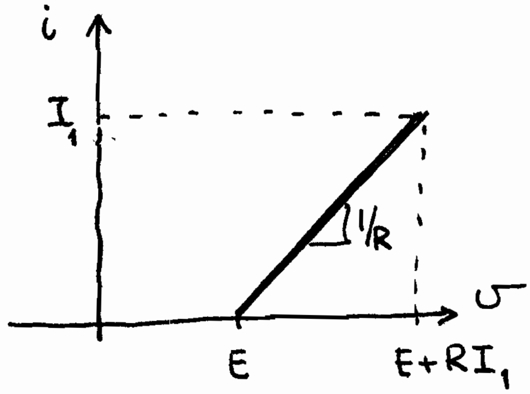
$$i \equiv i_D$$

risulta

$$0 \leq i < I_1$$

Dalla (1) risulta poi

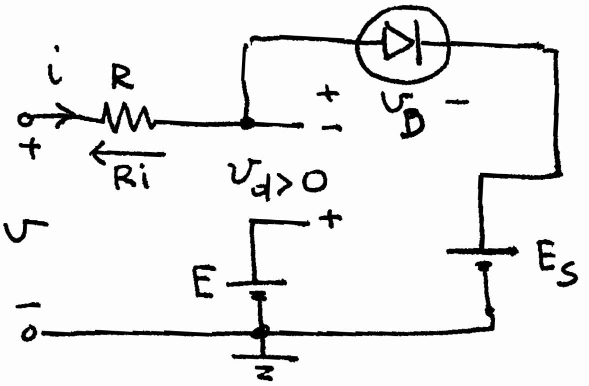
$$E \leq v < RI_1 + E$$



Si noti che I_1 risulta essere $\gg 1$ e quindi la regione lineare si estende per valori di tensione sufficienti per ogni uso pratico.

6) continuazione:

Zona di saturazione positiva



$(E < E_S)$

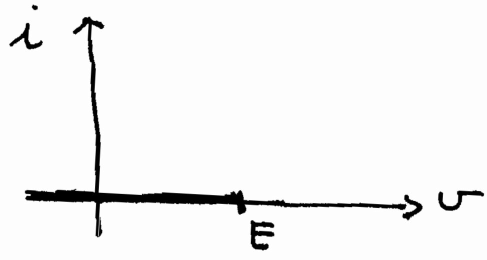
$$v_d = E - (v - Ri) = E - v + Ri > 0$$

$$v < E + Ri$$

Ora v_D è data da:

$$v_D = v - Ri - E_S < (E + Ri) - Ri - E_S = E - E_S < 0$$

Quindi $v_D < 0$ e quindi segue $i \equiv i_D = 0$, $v < E$



In totale: