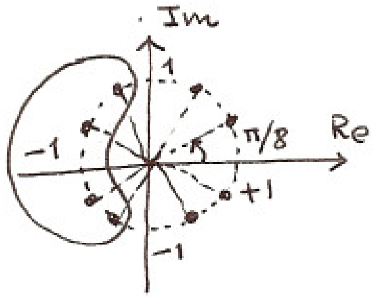
Si consideri il seguente modulo del coeff. di trasmissione:

$$|t(j\omega)|^2 = \frac{1}{1+\omega^8}$$

Si ricavi la corrispondente funzione (stabile) $t(p)$



La generatrice del modulo è

$$t(p)\,t(-p) = \frac{1}{1+p^8}$$

Devo trovare le radici di

$$p^8 + 1 = 0$$

e prendere solo quelle nel semipiano di sinistra

$$p^8 = -1$$

$$p_k = -\operatorname{sen}\varphi_k + j\cos\varphi_k$$

$$\varphi_k = \frac{\pi}{2n}(2k-1) \qquad k = 1, 2, \ldots n \quad \downarrow n=4$$

$$\varphi_k = \frac{\pi}{8}(2k-1) \qquad k = 1, 2, 3, 4$$

$$\varphi_1 = \frac{\pi}{8} \qquad \varphi_2 = \frac{\pi}{8}\times 3 \qquad \varphi_3 = \frac{\pi}{8}\times 5 \qquad \varphi_4 = \frac{\pi}{8}\times 7$$

$$p_{1,4} = -0{,}38 \pm j\,0{,}92$$

$$p_{2,3} = -0{,}92 \pm j\,0{,}38$$

Si hanno due fattori di secondo grado del tipo

$$\left[p-(-\operatorname{sen}\varphi+j\cos\varphi)\right]\left[p-(-\operatorname{sen}\varphi-j\cos\varphi)\right]=$$

$$=(p+\operatorname{sen}\varphi)^2+\cos^2\varphi = p^2+2p\operatorname{sen}\varphi+1$$

e quindi:

$$t(p)=\frac{1}{(p^2+0{,}76p+1)(p^2+1{,}84p+1)}$$

Per la successiva sintesi occorre una precisione maggiore:

$$(p^2+0{,}765\,366\,865p+1)(p^2+1{,}847\,759\,065p+1)$$

Sintesi del filtro LC bicaricato

$$t(p) = \frac{1}{(p^2+0{,}76p+1)(p^2+1{,}84p+1)} = \frac{f(p)}{g(p)} = \frac{1}{g(p)}$$

$f(p) = 1$

$g(p) = p^4 + 2{,}613\,125\,930\,p^3 + 3{,}414\,213\,562\,p^2 +$
$\quad + 2{,}613\,125\,930\,p + 1$

Notare che $t(0) = 1$ e quindi $R_u = 1$

$g(p)$ può essere calcolato con il comando Matlab

$w = \mathrm{conv}(u, v)$

Calcolo $\rho(p) = \dfrac{h(p)}{g(p)}$

- Eq. caratteristica:

  $h(p)h(-p) + f(p)f(-p) = g(p)g(-p)$

  $h(p)h(-p) = g(p)g(-p) - f(p)f(-p) = 1 + p^8 - 1 = p^8$

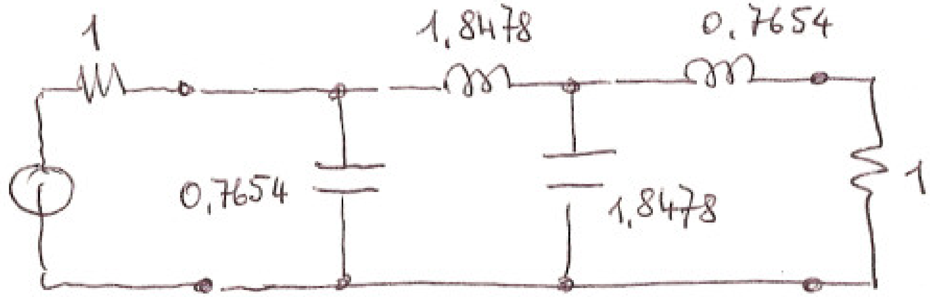
  $h(p) = \pm p^4$

- $Z_1(p) = \dfrac{1-\rho}{1+\rho} = \dfrac{g-h}{g+h}$

Scelgo $h(p) = p^4$

$$Z(p) = \frac{2{,}613\,125\,930\,p^3 + 3{,}414\,213\,562\,p^2 + 2{,}613\,125\,930\,p + 1}{2p^4 + 2{,}613\,125\,930\,p^3 + 3{,}414\,213\,562\,p^2 + 2{,}613\,125\,930\,p + 1}$$

	Z	Y	
	$2p^4 + 2.613125930p^3 + 3.414213562p^2 + 2.613125930p + 1$	$2.613125930p^3 + 3.414213562p^2 + 2.613125930p + 1$	
	$2p^4 + 2.613125930p^3 + 2p^2 + 0.765366865p$		$0.7654\,p$
	" " $1.414213562p^2 + 1.847759065p + 1$	$2.613125930p^3 + 3.414213562p^2 + 1.847759065p$	
$1.8478p$		" " $0.765366864p + 1$	$1.8478\,p$
	$1.414213562p^2 + 1.847759065p$		
	1		
$0.7654p + 1$		$0.76536686p + 1$	
		" "	



• Nota:

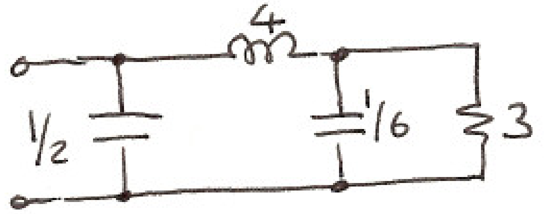
Le divisioni sono facilitate usando il comando Matlab

$[q, r] = \mathrm{deconv}(v, u) \qquad \left(\frac{v}{u} = q + \frac{r}{u} \right)$

SOLUZIONI – DIVISIONI SUCCESSIVE

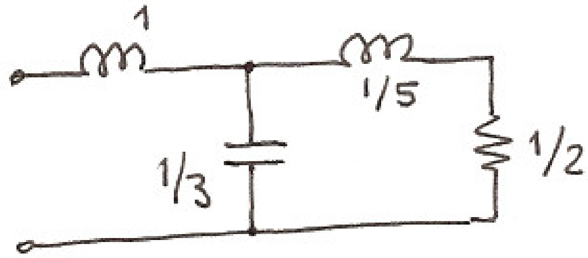
a)

Z	s^3+2s^2+2s+1	$2s^2+4s+3$	Y
	$s^3+2s^2+\frac{3}{2}s$	$2s^2+4s$	$\frac{1}{2}s$
	" " $0{,}5s+1$	" " 3	
$4s$	$0{,}5s+1$		$\frac{1}{6}s+\frac{1}{3}$
	" "		



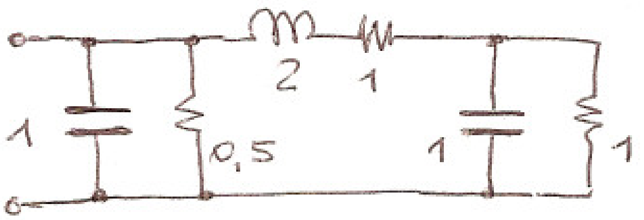
b)

Y	$2s^3+5s^2+36s+15$	$2s^2+5s+30$	Z
	$2s^3+5s^2+30s$	$2s^2+5s$	s
	" " $6s+15$	" " 30	$\frac{1}{5}s$
	$6s$		
$\frac{1}{3}s$	" 15		
2			



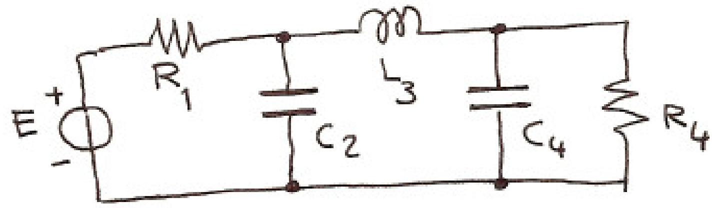
c)

Z	$2s^3+7s^2+9s+5$	$2s^2+3s+2$	Y	
	$2s^3+3s^2+2s$		$s+2$	I quoziente
	" $4s^2+7s+5$			
	$4s^2+6s+4$			
	" $s+1$	$2s^2+2s$		
		" $s+2$		
II quoz. $2s+1$		$s+1$		
		" 1	$s+1$	III quoz.
	$s+1$			
	" "			



NORMALIZZAZIONE (SCALING)

Si consideri il circuito seguente



$R_1 = R_4 = 75\,\Omega$, $C_2 = C_4 = 200$ pF , $L = 3\,\mu$H

Si determinino le costanti di normalizzazione R_0 e ω_0 in modo che, nel circuito normalizzato risulti $r_1 = r_4 = 1\,\Omega$, $c_2 = c_4 = 1$ F

(Fonte : Temes - La Patra, p. 20)

R: $R_0 = 75\,\Omega$

$$c = C R_0 \omega_0 = 1 \longrightarrow \omega_0 = \frac{1}{C R_0} = \frac{1}{200 \times 10^{-12} \times 75} =$$

$$= 66,67 \text{ Mrad/s}$$

$$l_3 = L_3 \omega_0 / R_0 = \frac{3 \times 10^{-6} \times 66.67 \times 10^6}{75} = \frac{200}{75} = \frac{8}{3} \text{ H}$$

Rete normalizzata :